PION DOUBLE CHARGE EXCHANGE ON NUCLEI AND INELASTIC RESCATTERINGS

A.B.Kaidalov, A.P.Krutenkova

Institute of Theoretical and Experimental Physics
Moscow, 117259, Russia

Abstract

The inclusive pion double charge exchange (DCX) process on nuclei is considered in the framework of the Gribov–Glauber approach. It is shown that inelastic rescatterings related to production of two (and more) pions in the intermediate state give an important contribution to the process of DCX at kinetic energies above $\sim 0.5 \ GeV$. This mechanism dominates at energies $\gtrsim 1 \ GeV$ and allows to explain the weak energy dependence of the DCX cross section observed experimentally [1].

The pion double charge exchange process is an interesting probe [2] of nuclear structure and of a hadron dynamics as it involves interaction with at least two nucleons of a nucleus. The standard DCX mechanism corresponds to two sequential single charge exchanges of pion on nucleons of a nucleus (fig. 1(a)). This model gives a reasonable description of experimental data on DCX at kinetic energies of incident pions $T_0 \lesssim 0.5 \ GeV$ and predicts [3] a strong decrease of a small angle DCX at higher energies. However recent measurements of DCX at energies up to 1.1 GeV performed at ITEP showed [1] that the differential cross section of this process decreases with energy rather weakly and exceeds the theoretical prediction [4] at the highest energy by an order of magnitude. Thus other mechanisms of DCX are needed to explain the observed energy dependence.

In this note we will show that inelastic rescatterings of the type shown in fig. 1(b),(c) give an important contribution to the cross section of DCX at energies $T_0 > 0.5 \ GeV$ and allow to understand the experimentally observed pattern of the energy dependence for this reaction. In the following we shall use the Gribov formalism for a description of rescatterings on nuclei [5]. We

will also compare our results with the predictions of a model based on meson exchange currents.

The inclusive cross section of DCX has been measured in the reaction $A(\pi^-, \pi^+)X$ for ^6Li and ^{16}O nuclei and at energies $T_0 = 0.6$, 0.75 and 1.1 GeV ($\langle\theta\rangle \approx 5^0$) [1]. The kinematics of the process was chosen in such a way as to forbid production of an extra pion, $\Delta T = T_0 - T \leq m_\pi$, where T is the kinetic energy of the produced pion. In this respect it differs strongly from the kinematics studied in Ref.[6]. The energy of recoil nucleons has not been measured and it is necessary to integrate over an energy of the produced neutrons in the kinematical region indicated above and to average over Fermi motion of the protons participating in the reaction (Fig.1). Note that the masses of the intermediate hadronic states, H, which can be produced in the reaction $\pi^- p \rightarrow Hn$, increase with incident energy as $M_H^2 \sim 2m_N E_0$ (even in the kinematical region of the experiment).

The most general diagram for pion DCX on nuclei is shown in fig. 2. In the Gribov approach [5] an integration over the longitudinal momenta of nucleons p_i, p_i' ($i = 1, 2$) in fig. 2 can be rewritten as an integral over $M_H^2 = (p_\pi + p_1 - p_1')^2 = (p_\pi' + p_2' - p_2)^2$, square of invariant mass of the intermediate hadronic system, which is taken along the contour C in the complex M_H^2 plane, shown in fig. 3. The integral at large M_H^2 is effectively cut due to a smallness of the Fermi momentum and experimental limits on kinetic energies of final nucleons $T_1' + T_2' \leq \Delta T$ ($M_H^2 \lesssim 2E_0 (\Delta T \cdot m_N)^{1/2}$). At large enough energies it is convenient to deform the contour C to C' (fig. 3) and to drop a contribution of a large circle if the amplitude decreases with M_H^2 faster than $1/M_H^2$ at large M_H^2. The asymptotic behaviour at large M_H^2 is determined by the corresponding j–plane singularity in the t channel. In our case it corresponds to an exchange of a state with the charge $Q = 2$ (isospin $I \geq 2$). There are no known Regge poles with such "exotic" quantum numbers and it is reasonable to assume that the amplitude decreases fast enough at large M_H^2. In this case the integration over C' corresponds to taking a discontinuity over the physical cut on a real axis. In this way we get the diagrams of fig. 1(a),(b) as the pole contributions and the diagram of fig. 1(c) represents an intermediate 2π state. Contributions from intermediate 3π, 4π ... states should be also taken into account though their contribution can be small. These considerations, strictly speaking, are valid at high energies $E_0 \gg 1$ GeV. For energies $E_0 \lesssim 1$ GeV a contribution from a circle in M_H^2 plane is not vanishingly small in general. In our estimates in the following we shall neglect this contribution.

Experimental data [7] on cross sections of different pion charge–exchange

processes on the proton in the energy region $0.3\ GeV < T_0 < 1.3\ GeV$ are compiled in fig. 4. It follows from this figure that η^0 production gives a relatively small correction to the diagram of fig. 1(a), while production of two pions is a very important competing mechanism (especially for $T_0 > 0.5\ GeV$).

Note that for forward scattering on nuclei the inelastic corrections to Glauber model are important only at high energies $E_0 \gtrsim 5\ GeV$. They are not very large and are approximately equal $\sim 20 \div 30\%$ of elastic correction [8] (but important for agreement with experiment). It is interesting that in the DCX case inelastic rescatterings can be dominant already at much smaller energies $T_0 \gtrsim 1\ GeV$.

Now we shall estimate the contributions of inelastic rescatterings (see Tables 1,2 below) using experimental data on particle production in $\pi^- p$ interactions. Diagrams of fig. 1 in general interfere in cross section. However if the $\pi\pi$–production amplitude is dominated by π exchange (fig. 5) then there will be no interference between diagrams of fig. 1(a) and fig. 1(c) due to difference in spin structure of nucleon vertices. Thus in the following we shall neglect interference terms and shall write the forward cross section of DCX in the following form

$$\frac{d\sigma_{DCX}}{d\Omega} = \frac{d\sigma_{DCX}^{\pi^0}}{d\Omega} + \frac{d\sigma_{DCX}^{\eta^0}}{d\Omega} + \frac{d\sigma_{DCX}^{\pi^+\pi^-}}{d\Omega} + \frac{d\sigma_{DCX}^{\pi^0\pi^0}}{d\Omega} \tag{1}$$

where the cross section $d\sigma_{DCX}^{H}/d\Omega$ denote square of amplitude of the diagrams of fig. 1 for an intermediate state H.

For single particle intermediate state $(H = \pi^0,\ \eta^0)$

$$\frac{d\sigma_{DCX}^{H}}{d\Omega} \sim \left(\int \frac{d\sigma_{\pi^- p \to Hn}}{dt_1} dt_1 \right)^2 . \tag{2}$$

This is also true for s–wave $\pi\pi$–nonresonance intermediate states, which dominates $\pi\pi$ production at low energies. So we can write

$$\frac{d\sigma_{DCX}}{d\Omega} = \frac{d\sigma_{DCX}^{\pi^0}}{d\Omega} \left(1 + \sum_H \Delta_H \right) \tag{3}$$

where

$$\Delta_H = \left(\int \frac{d\sigma_{\pi^- p \to Hn}}{dt_1} dt_1 \right)^2 \bigg/ \left(\int \frac{d\sigma_{\pi^- p \to \pi^0 n}}{dt_1} dt_1 \right)^2 . \tag{4}$$

For production of two (or more) pions the quantity, which enters into (4), in general does not coincide with the cross section of the corresponding reaction

integrated over the kinematical variables of pions because we consider nondiagonal transition $\pi^- \to \pi^+$. This is especially clear from the diagram of fig. 1(c) in the pion exchange model, i.e. fig. 6. In this case an imaginary part of backward $\pi^- \pi^+$ amplitude (not forward elastic $\pi^- \pi^+$ amplitude) enters into the diagram. A difference between forward and backward amplitudes is connected to an interference between $\pi\pi$ states with odd and even angular momenta and can be taken into account if a phase shift analysis is used. However this interference is small for masses of $\pi\pi$ states $\lesssim 600 \ MeV$, which give a dominant contribution at the investigated region of energies. The contribution of π^0 into $d\sigma_{DCX}/d\Omega$ ($d\sigma_{DCX}^{\pi^0}/d\Omega$ in Eqs.(1),(3)) has been calculated before in Ref.[1] for ^{16}O (see Table 2 and abbreviation $SSCX$ model in fig. 7), using a Monte Carlo cascade model [4], developed to study pion induced multichannel reactions at pion energies above 0.4 GeV.

Corrections Δ_H due to different intermediate states calculated using experimental data [7] on corresponding processes are given in Table 1 and their total contribution is shown in fig. 7. It follows from Table 1 that the main contribution into DCX is given by the process $\pi^- p \to \pi^+ \pi^- n$. An account of all corrections $\sum_H \Delta_H$ ($SSCX + GR1$ in fig. 7) leads to a reasonable agreement with the experiment (except values at $T_0 = 0.6 \ GeV$ where these corrections are less reliable).

$T_0, \ GeV$	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
Δ_η	–	–	0.02	0.15	0.03	0.02	0.12	0.14	0.09
$\Delta_{\pi^0\pi^0}$	0.01	0.03	0.08	0.32	0.21	0.17	0.47	0.30	0.21
$\Delta_{\pi^+\pi^-}$	0.08	0.32	0.48	2.22	2.21	2.28	5.18	6.00	5.32
$\sum_H \Delta_H$	0.09	0.35	0.58	2.69	2.45	2.47	5.77	6.44	5.62

Table 1. The quantities Δ_H (see Eqs.(3),(4)) of the inelastic rescatterings for $H = \eta^0, \ \pi^0\pi^0, \ \pi^+\pi^-$.

In this simple estimate we have not yet taken into account experimental limitations on energy transferred to produced neutrons and to spectator nucleons, $\Delta T^{\mathrm{max}} = 140$ (or 80) MeV, used in the experiment [1]. This condition influences the regions of integrations on t_1 and t_2 (see fig. 2) in Eq.(4) and can lead to some distortion of the results obtained above if the reaction $\pi^- p \to \pi^0 n$ and other processes $\pi^- p \to Hn$ have different t dependences. We

have estimated an importance of this effect using the following simple model:

a) We considered a configuration when final neutrons have approximately equal kinetic energies $T'_1 \simeq T'_2 \leq \Delta T^{\mathrm{max}}/2$. This leads to the following limits on invariant momentum transfer $|t_i| \leq 2m_N (\Delta T^{\mathrm{max}}/2)$.

b) The most important intermediate states $\pi^+\pi^-$ and $\pi^0\pi^0$ were considered.

c) The quantities $\Delta_{\pi^+\pi^-}$ and $\Delta_{\pi^0\pi^0}$ in Eqs.(3),(4) were replaced by

$$\Delta'_{\pi^+\pi^-} = \left(\int dM \int_{t_{1\,\min}(M)}^{t_{1\,\max}^{\mathrm{exp}}} \frac{d^2\sigma_{\pi^+\pi^-}(M, t_1)}{dM \, dt_1} dt_1 \middle/ \int_0^{t_{1\,\max}^{\mathrm{exp}}} (d\sigma_{\pi^0}/dt_1) dt_1 \right)^2 , \qquad (5a)$$

$$\Delta'_{\pi^0\pi^0} = \left(\sigma^{tot}_{\pi^0\pi^0} \middle/ \sigma^{tot}_{\pi^+\pi^-} \right)^2 \cdot \Delta'_{\pi^+\pi^-} \qquad (5b)$$

The quantity $\Delta'_{\pi^+\pi^-}$ has been calculated using experimental data on distributions on $\cos\theta(n, p)$ and $M^2(\pi^+\pi^-)$ for the reaction $\pi^-p \to \pi^+\pi^-n$ given in Ref.[7c] for the interval of energies we are interested in. Differential cross section $d\sigma/dt$ for the reaction $\pi^-p \to \pi^0 n$ has been taken from the result of the phase shift analysis (see [7(d)]). The results are given in Table 2 and in fig. 7 ($d\sigma_{DCX}/d\Omega$ and $d\sigma'_{CDE}/d\Omega$ in Table 2 and $SSCX + GR2$ in fig. 7). We see that taking into consideration the experimental limitations on ΔT leads to substantial decrease of rescattering correctons at $T_0 = 0.6 \ GeV$ and their increase at $1.12 \ GeV$.

$T_0, \ GeV$	0.6	0.75	1.12
$d\sigma^{\pi^0}_{DCX}/d\Omega, \ \mu b/sr$	125.0 (23.0)	10.4 (2.4)	3.1 (0.31)
$1 + \sum_H \Delta_H$	1.58	4.21	7.28
$d\sigma_{DCX}/d\Omega, \ \mu b/sr$	197.5 (36.3)	43.8 (10.1)	22.6 (2.3)
$1 + \Delta'_{\pi^+\pi^-} + \Delta'_{\pi^0\pi^0}$	1.11 (1.05)	2.75 (1.69)	12.3 (7.81)
$d\sigma'_{DCX}/d\Omega, \ \mu b/sr$	139.0 (24.1)	28.6 (4.06)	38.3 (2.42)
$d\sigma^{\mathrm{exp}}/d\Omega, \ \mu b/sr$	59.6 ± 7.4 (22.6 ± 4.5)	43.3 ± 5.5 (12.2 ± 3.2)	26.6 ± 8.9 (8.3 ± 5.5)

Table 2. The cross section of the inclusive pion DCX on ^{16}O measured in experiment [1b] $(d\sigma^{\exp}/d\Omega)$ and calculated:
(a) in Ref.[1] for $H = \pi^0$ using an algorithm of Ref.[4] $(d\sigma^{\pi^0}_{DCX}/d\Omega)$;
(b) according to Eqs.(1)–(4) for $H = \pi^0$, η^0, $\pi^0\pi^0$, $\pi^+\pi^-$ $(d\sigma_{DCX}/d\Omega)$;
(c) according to Eqs.(1)–(3),(5) for $H = \pi^0$, $\pi^0\pi^0$, $\pi^+\pi^-$ taking into account the kinematical region ΔT of the experiment [1] $(d\sigma'_{DCX}/d\Omega)$.
All the differential cross sections presented are integrated over the two regions of ΔT: from 0 to 140 MeV and to 80 MeV (in parentheses).

An overall agreement between experimental data and predictions of the model with inelastic rescatterings is quite satisfactory taking into account that we have made several simplifying assumptions discussed above. In particular in the region of energies $T_0 \lesssim 1\ GeV$ there can be a substantial contribution from the production of baryon resonances in the direct channel of the process $\pi^- p \to \pi^+\pi^- n$ and an interference between contributions of different diagrams of fig. 1 is possible.

Let us discuss a relation between inelastic rescatterings considered in our paper and other mechanisms which have been proposed for the DCX process. One of the most popular mechanisms is a model of meson exchange currents [10], which corresponds to the diagrams of fig. 8 for $\pi^- pp \to \pi^+ nn$ transition. The black dots in this figure are point–like vertices. Comparing the diagram of fig. 8(a) with the diagram of fig. 6 we see that both take into account $\pi^-\pi^+ \to \pi^+\pi^-$ transitions, but the diagram of fig. 8 takes into account a real part of this amplitude, which is small for soft pions, while the diagram of fig. 6 is related to an imaginary part of this amplitude, which is close to a maximum allowed by unitarity in the region of $M_{\pi\pi} \sim 500 - 800\ MeV$. Inclusion of the diagram of fig. 8(b) leads to an extra decrease of the total contribution of this mechanism. A recent calculation for the exclusive DCX reaction [11], which takes into account, besides the diagram of fig. 1(a), the meson exchange currents (fig. 8) shows that the contribution of the last mechamism is too small to account for the experimentally observed cross sections of DCX at energies above $0.5\ GeV$.

Another possible mechanism for DCX is an interaction of a pion with a correlated pair of nucleons (see e.g. [12]), which can be represented at the quark level by the diagram of fig. 9. The characteristic feature of such mechanism (as well as of meson exchange currents), is that only neighboring nucleons of a nucleus can interact in this way. On the contrary, inelastic rescatterings of fig. 1(b),(c) can involve nucleons separated by large longitudinal distances (at

least at high energies). This leads to a difference in A dependence of the DCX processes for mechanisms mentioned above.

If our assumption that integration over M_H^2 converges fast enough is correct then at energies much higher than 1 GeV the cross section of DCX will be governed by the corresponding ρ and π exchanges in t_i channels. In this case the cross section should decrease at least as fast as $s^{2[(2\alpha_\rho(0)-1)-1]} \approx s^{-2}$ (see, e.g. review lectures [13]) and its rather slow energy dependence at energies below 1.1 GeV should change to a much faster decrease. What can be expected in the region of energies $T_0 \sim 2-5~GeV$, the closest one to the experiment [1]? The theoretical estimates used above become more reliable at these energies because corrections to theoretical formulae decrease with energy and the one pion exchange model gives an adequate description of the main features of $\pi \to 2\pi$ process here [14]. The available data on the total cross section of the reaction $\pi^- p \to \pi^+\pi^- n$ [15] and $\pi^- p \to \pi^0 n$ [7(d)] show that the value of inelastic rescatterings at these energies is still rather large ($\Delta_{\pi^+\pi^-} \gg 1$). Thus we expect large deviations from the standard DCX mechanism also in this energy region. The effect can be estimated using the method proposed in this paper.

It is highly desirable to extend measurements of DCX to these energies taking into account that DCX gives a unique possibility to observe a large contribution of inelastic rescatterings and to understand better a mechanism of this process. (This is in contrast to the case of elastic scattering of hadrons on nuclei where inelastic rescatterings lead to rather small effects.) The value of the inclusive pion DCX cross section integrated over the region $\Delta T = 0 \div 140~MeV$ is expected to be high enough ($\sim 20~\mu b/sr$ at 2 GeV) to be detected in experiment. The experimental study of the A dependence could give additional information in order to distinguish between the sequential mechanism and the pion interaction with correlated pairs of nucleons.

References

[1] (a) B.M.Abramov et al., Yad. Fiz. **59**, 399 (1996) (English translation: Phys. Atomic Nucl. **59**, 376 (1996));
(b) B.M.Abramov et al., Few–Body Systems Suppl. **9**, 237 (1995).

[2] H.Clement, Progr. Part. Nucl. Phys. **29**, 175 (1992);
M.B.Johnson, and C.L.Morris, Annual Rev. Nucl. Part. Sci. **43**, 165 (1993).

[3] E.Oset, and D.Strottman, Phys. Rev. Lett. **70**, 146 (1993).

[4] M.J.Vicente Vacas, M.Kh.Khankhasayev, and S.G.Mashnik, Preprint FTUV/94–73 University of Valencia 1994.

[5] V.N.Gribov, ZhETF **56**, 892 (1969) (English translation: Sov. Phys. JETP **29**, 483 (1969)).

[6] J.–B.Jeanneret, M.Bogdanski, and E.Jeannet, Nucl. Phys. A **350**, 345 (1980).

[7] (a) A.D.Brown et al., Nucl. Phys. B **153**, 89 (1979);
(b) D.M.Manley et al., Phys. Rev. D **30**, 904 (1984);
(c) A.D.Brody et al., Phys. Rev. D **4**, 2693 (1971);
(d) R.A.Arndt et al., Phys. Rev. C52, 2120 (1995).

[8] V.V.Anisovich, L.G.Dakhno, and P.E.Volkovitskii, Phys. Lett. B **42**, 224 (1972);
A.B.Kaidalov, and L.A.Kondratyuk, Nucl. Phys. B **56**, 90 (1973);
V.A.Karmanov, and L.A.Kondratyuk, Pisma ZhETF **18**, 451 (1973).

[9] (a) S.A.Wood et al., Phys. Rev. C46, 1903 (1992);
(b) R.G.Burleson, in *Pion–Nucleus Double Charge Exchange*, Proceedings of the Second LAMPF Workshop on Pion–Nucleus Double Charge Exchange, Los Alamos, New Mexico, USA, p.79. Singapore, World Scientific 1990.

[10] J.–F.Germond, and C.Wilkin, Lett. Nuovo Cim. **13**, 605 (1975);
M.R.Robilotta, and C.Wilkin, J. Phys. G: Nucl. Phys. **4**, L115 (1978).

[11] L.Alvarez–Ruso, and M.J.Vicente Vacas, J. Phys. G: Nucl. Part. Phys. **22**, L45 (1996).

[12] G.A.Miller, Phys. Rev. C **35**, 377 (1987).

[13] A.B.Kaidalov, in *Survey in High Energy Phys.*, (Overseas Publ. Assoc., Amsterdam, 1996), v.9, p.143.

[14] K.G.Boreskov, A.B.Kaidalov, and L.A.Ponomarev, Yad. Fiz. **17**, 1285 (1973).

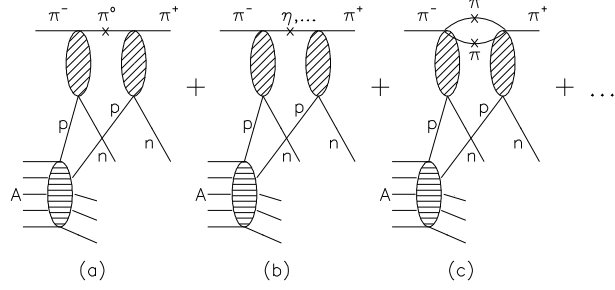[15] B.G.Reynolds et al., Phys. Rev. **184**, 1424 (1969).

Figure 1: Diagrams contributing to pion double charge exchange on nuclei: (a) sequential single charge exchanges (SSCX), i.e. standard mechanism (elastic rescattering), (b) quasielastic rescatterings, (c) inelastic rescatterings.
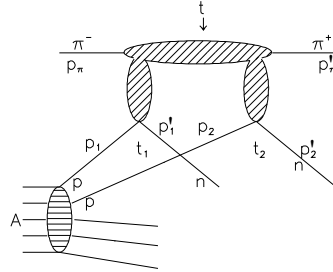

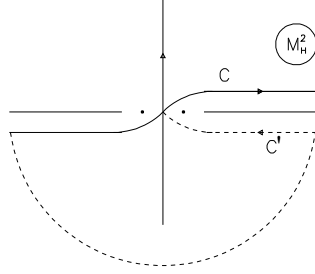Figure 2: Pion double charge exchange on nucleus (the most general diagram).

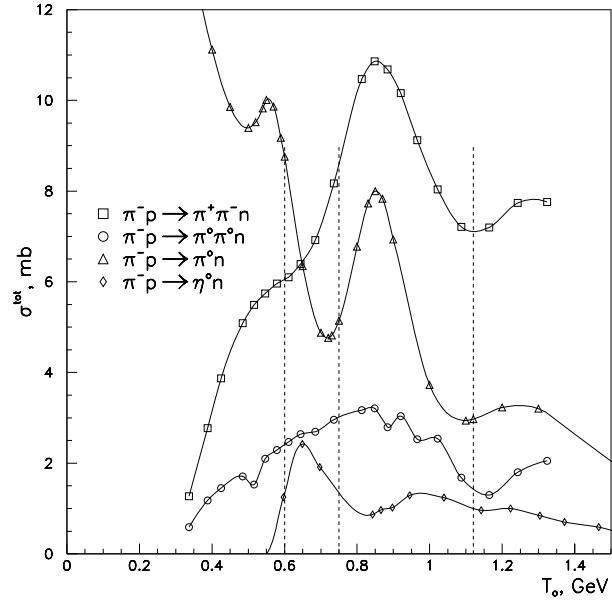Figure 3: Contour of integration in complex M_H^2 plane.

Figure 4: Energy dependence for total cross sections of $\pi^- p \to Hn$ processes. Vertical dashed lines mark three values of the incident π^- energy in the experiment [1].
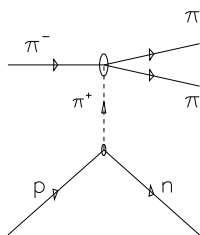
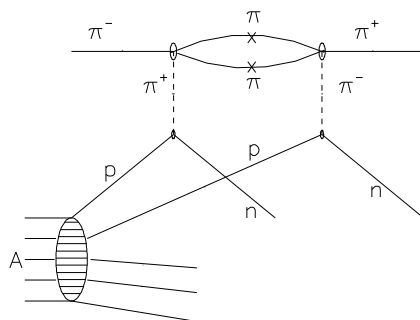Figure 5: The process $\pi^- p \to \pi\pi n$ in one pion exchange model.

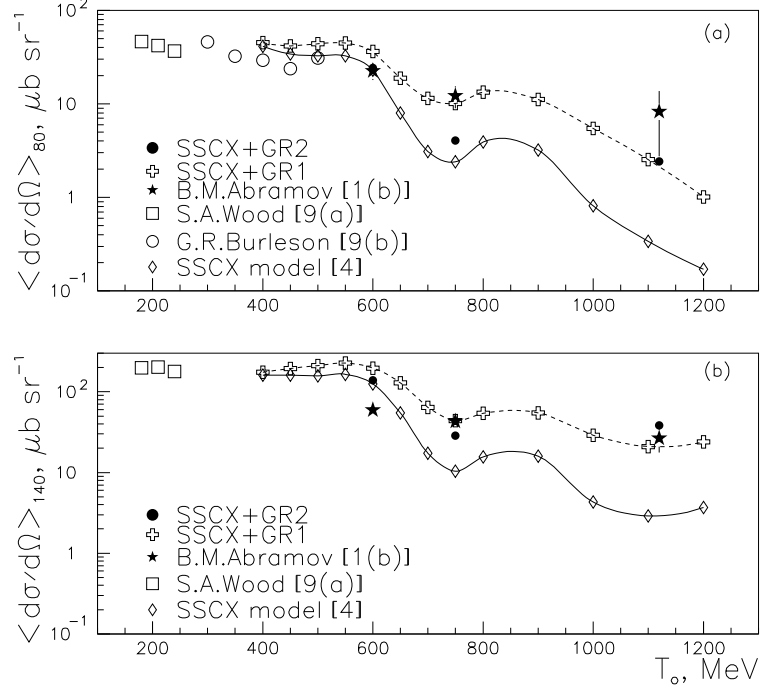Figure 6: Pion double charge exchange in one pion exchange model.

Figure 7: Experimental cross sections of the inclusive forward pion double charge exchange on ^{16}O integrated over the region $\Delta T = 0 - 80$ MeV (a) and $\Delta T = 0 - 140$ MeV (b). The theoretical calculations are performed in the framework of cascade model (SSCX) and with the inelastic Glauber rescattering corrections (GR).
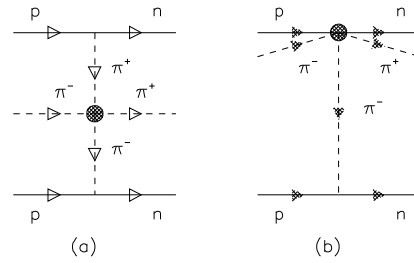

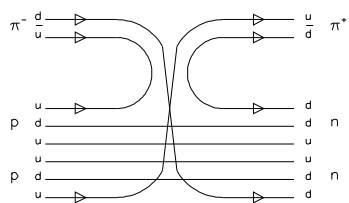Figure 8: Pion double charge exchange in the meson exchange currents mechanism.

Figure 9: Quark diagram for pion double charge exchange.